

GUT Scale Threshold Corrections
in a Complete Supersymmetric SO(10) Model:
 $\alpha_s(M_Z)$ **vs. Proton Lifetime**

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Abstract

We show that one loop GUT scale threshold corrections to gauge couplings are a significant constraint on the GUT symmetry breaking sector of the theory. The one loop threshold corrections relate the prediction for $\alpha_s(M_Z)$ to the proton lifetime. We have calculated these corrections in a new complete SO(10) SUSY GUT. The results are consistent with the low energy measurement of $\alpha_s(M_Z)$. We have also calculated the proton lifetime and branching ratios in this model. We show that proton decay rates provide a powerful test for theories of fermion masses.

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I. INTRODUCTION

It has been known for some time now that Supersymmetric [SUSY] Grand Unified Theories [GUTs] [1]

- suppress the non-SUSY GUT prediction for proton decay, via heavy gauge boson exchange, by increasing the GUT scale, and
- lead to a 10% increase (for one pair of Higgs doublets) in the GUT prediction for the weak mixing angle, $\sin^2 \theta_W$, compared to non-SUSY GUTs.

Now, since $\sin^2 \theta_W$ is known to better than 1% accuracy, one naturally uses the values of the two low energy parameters, the fine structure constant, α , and $\sin^2 \theta_W$ as input to determine the GUT coupling, α_G , and the GUT scale, M_G , and predicts the value of the strong coupling, α_s . [Note, all low energy parameters are typically evaluated at the scale, M_Z .] This prediction is in excellent agreement with the low energy data [2–4]. The global fit to α_s from measurements at all energies [2] gives $\alpha_s = 0.117 \pm 0.005$ with low (high) energy measurements favoring values of $\alpha_s \leq (\geq) 0.12$. A global fit to all electroweak data [3] gives $\alpha_s = 0.127 \pm 0.005 \pm 0.002 \pm 0.001$. In comparison, SUSY GUTs predict [4] $\alpha_s = 0.127 \pm 0.005 \pm 0.002$, where the first error is the uncertainty in the low energy sparticle spectrum, and the second, the top and Higgs masses. Note that the non-supersymmetric GUT prediction [5] $\alpha_s = 0.073 \pm 0.001 \pm 0.001$ is clearly ruled out.

In addition, SUSY GUTs contain new sources of proton decay (dimension 5 operators) mediated by color triplet Higgs fermions [6,7]. They typically lead to the dominant decay mode $p \rightarrow K^+ \bar{\nu}$ with the lifetime, $\tau_p \propto \tilde{M}_t^2$, where \tilde{M}_t is an effective color triplet mass.

These results are valid for any grand unified symmetry group, as long as $SU(3) \times SU(2) \times U(1)$ is embedded in an unbroken simple subgroup above the GUT scale. Consideration of fermion masses, however, leads us to the special grand unified symmetry group $SO(10)$ [8]. It is the unique group which combines all the fermions of one family $\{u, d, e, \nu\}$ into one irreducible 16 dimensional representation with the addition of only one electroweak singlet state, $\bar{\nu}$, the so-called right-handed neutrino. Consequently, $SO(10)$ Clebschs can be used

to relate all charged fermion mass matrices. In this context, a paper by Anderson et al. [9][Paper I] showed how all charged fermion masses and mixing angles can be reasonably described in terms of four dominant effective mass operators at M_G . Hall and one of us (S.R.) [10] [Paper II] then showed how to extend the effective theory at M_G to a complete SO(10) SUSY GUT valid up to the Planck or string scales, M . That theory included several sectors necessary for – (1) GUT symmetry breaking, (2) doublet-triplet splitting (Higgs sector), (3) charged fermion masses (fermion mass sector) and also (4) giving a large mass to the electroweak singlet neutrinos (neutrino mass sector). It contained sufficient symmetry to be “natural.” Hence, it included all operators consistent with the symmetries and only contained the SO(10) states¹ —

$$(n_{16} + 3) \mathbf{16} + n_{16} \bar{\mathbf{16}} + n_{10} \mathbf{10} + n_{45} \mathbf{45} + n_{54} \mathbf{54} + n_1 \mathbf{1} \quad (1)$$

with $n_{16}, n_{10}, n_{45}, n_{54}, n_1$ specific integers. Finally, the effective mass operators of paper I were recovered upon integrating out states with mass greater than M_G .

In any complete GUT, the prediction for α_s receives corrections at one loop from thresholds at the weak scale, including the sparticle spectrum, and at the GUT scale from states with mass of order M_G . The dominant contribution at M_G comes from states in the symmetry breaking and Higgs sectors.

The Higgs sector, on the other hand, also affects the proton decay rate. The effective color triplet Higgs fermion mass, \tilde{M}_t , must be significantly larger than M_G in order to increase the proton lifetime. The Higgs doublets on the other hand must remain massless at the GUT scale.

In this paper,

- we show that the threshold corrections due to doublet-triplet splitting, in the simplest models, increases the predicted value for $\alpha_s(M_Z)$. Thus, while the GUT predictions for α_s and τ_p are unrelated at the tree level, at one loop the two effects are coupled. One cannot

¹These two features are consistent with a stringy origin to such a model [11].

suppress proton decay without at the same moment increasing the prediction for α_s . We then show that the model of paper II is ruled out by this effect.

- We also present a new complete SUSY GUT which appears to be consistent with all low energy data at the 1σ level [12]. This model has in fact a much simpler GUT symmetry breaking sector than that in paper II with fewer states. We present some typical results for proton decay rates in this model. In a future paper [13], we will present a more detailed study.

II. ONE LOOP THRESHOLD CORRECTIONS AT M_G .

One loop threshold corrections at M_G for gauge couplings are given by [14]

$$\alpha_i^{-1}(M_G) = \alpha_G^{-1} - \Delta_i \quad (2)$$

where Δ_i is the leading log threshold correction to α_i . Furthermore,

$$\Delta_i = \frac{1}{2\pi} \sum_{\zeta} b_i^{\zeta} \log \left| \frac{M_{\zeta}}{M_G} \right| \quad (3)$$

where the sum is over all superheavy particles and b_i^{ζ} is the contribution the superheavy particle ζ would make to the beta function coefficient b_i if the particle were not integrated out at M_G .

At one loop the definition of the GUT scale is somewhat arbitrary. In order to avoid large logarithms it must certainly be at the geometric mean of the heavy masses; otherwise we are free to choose its value. A particularly convenient choice is to define M_G as the scale where the two gauge couplings, α_i , $i = 1, 2$, meet. At this point $\Delta_1 = \Delta_2$ and we define

$$\tilde{\alpha}_G \equiv \alpha_1(M_G) = \alpha_2(M_G). \quad (4)$$

Then define

$$\epsilon_3 \equiv (\alpha_3(M_G) - \tilde{\alpha}_G)/\tilde{\alpha}_G, \quad (5)$$

i.e. the relative shift in α_3 at M_G . In general, a value of $\epsilon_3 \sim -(2 - 3\%)$ is needed to obtain $\alpha_s \sim 0.12$.

A. Formula for ϵ_3 in a general SO(10) theory

The most general SO(10) SUSY GUT we consider includes the complete multiplets listed in equation (1) with arbitrary values of $n_{16}, n_{10}, n_{45}, n_{54}, n_1$. These states can be decomposed into their $SU(3) \times SU(2) \times U(1)$ content by first considering the decomposition to SU(5) [see table I] and then the rest of the way [15].

We have derived a general formula for ϵ_3 in this case (see appendix 1) given by the expression (valid to lowest order in $\tilde{\alpha}_G$)

$$\epsilon_3 = \frac{\tilde{\alpha}_G}{2\pi} \sum_{\gamma} [(b_3^{\gamma} - b_2^{\gamma}) - \frac{1}{2}(b_2^{\gamma} - b_1^{\gamma})] \log \left| \det' \frac{M_{\gamma}}{M_G} \right| \quad (6)$$

where

$$\det' M_{\gamma} = \begin{cases} \det M_{\gamma} & \text{if } \gamma = t, g, w, s, \sigma \\ \frac{\det M_{\gamma}}{(M_{gauge}^{\gamma})^4} & \text{if } \gamma = q, u, e, x \\ \det M'_d & \text{if } \gamma = d \end{cases} \quad (7)$$

where M'_d is defined as the doublet mass matrix M_d with the massless Higgs doublets projected out.² Note, ϵ_3 only depends on the number of states in the theory through the mass matrices M_{γ} in each charge sector γ . Moreover, the effective determinants, defined above, explicitly take into account the special contributions of vector multiplets to the threshold corrections.

Putting in the values of b_i^{γ} for $i = 1, 2, 3$ into eqn. (6) we find

$$\begin{aligned} \epsilon_3 = & \frac{\tilde{\alpha}_G}{\pi} \left(\frac{3}{2} \log \left| \det' \frac{M_g}{M_G} \right| - \frac{3}{2} \log \left| \det' \frac{M_w}{M_G} \right| \right. \\ & + \frac{33}{10} \log \left| \det' \frac{M_s}{M_G} \right| - \frac{21}{10} \log \left| \det' \frac{M_{\sigma}}{M_G} \right| + \frac{9}{10} \log \left| \det' \frac{M_u}{M_G} \right| + \frac{3}{10} \log \left| \det' \frac{M_e}{M_G} \right| - \frac{6}{5} \log \left| \det' \frac{M_q}{M_G} \right| \\ & \left. - \frac{3}{5} \log \left| \det' \frac{M_d}{M_G} \right| + \frac{3}{5} \log \left| \det' \frac{M_t}{M_G} \right| \right). \end{aligned} \quad (8)$$

The dominant contribution to ϵ_3 comes from the GUT symmetry breaking and electroweak Higgs sectors with superspace potential $W_{sym\ breaking}$ and W_{Higgs} , respectively. An

²Our notation for the states discussed above is found in table II.

additional small contribution comes from the fermion mass sector. These sectors are by reasons of “naturalness” necessarily invariant under several U(1) symmetries (including an R symmetry). In appendix 2 we show that these symmetries impose stringent constraints on the form of ϵ_3 . In brief, ϵ_3 is implicitly a function of the vacuum expectation values [vevs] of fields in the symmetry breaking sector. These vevs transform under the U(1) and R symmetries. As a consequence of the invariance, we find

$$\epsilon_3 = f(\zeta_1, \dots, \zeta_m) + \frac{3\tilde{\alpha}_G}{5\pi} \log \left| \frac{\det \bar{M}_t}{M_G \det \bar{M}'_d} \right| + \dots \quad (9)$$

where the first term represents the contributions from $W_{sym\ breaking}$. It is only a function of U(1) and R invariant products of powers of vevs $\{\zeta_i\}$. The second term, coming from the Higgs sector, is discussed further in the next section and the ellipsis refers to the small additional contribution arising from the fermion mass sector. Note, \bar{M}_t , \bar{M}'_d only include those states, from $\mathbf{5}$ s and $\bar{\mathbf{5}}$ s of SU(5) contained in $W_{sym\ breaking}$ and W_{Higgs} , *which mix with the Higgs sector* (see the next section).

B. The dependence of ϵ_3 on the Higgs sector

We assume the Higgs sector includes any number of $\mathbf{10}$ s with, for simplicity, only one of these, say $\mathbf{10}_1$, coupling to light fermions. We also include the doublet-triplet splitting mechanism introduced by Dimopoulos-Wilczek [16]. Accordingly, the terms in the super-space potential relevant for doublet - triplet splitting are of the form

$$W_{d-t} = \bar{\mathbf{5}}_2 \left[a_1 \frac{3}{2} (B - L) \right] \mathbf{5}_1 - \bar{\mathbf{5}}_1 \left[a_1 \frac{3}{2} (B - L) \right] \mathbf{5}_2 + \sum_{i,j \geq 2} M_{ij} \bar{\mathbf{5}}_i \mathbf{5}_j \quad (10)$$

where $a_1 \frac{3}{2} (B - L)$ is the vev of the field A_1 in the 45 dimensional representation and $(B - L)$ is the [Baryon - Lepton number] charge matrix (see eqn. (14)). Thus, since the doublets in $\mathbf{5}_1, \bar{\mathbf{5}}_1 \subset \mathbf{10}_1$ have zero $B - L$, they remain massless. [Note, some of the $\bar{\mathbf{5}}_i, \mathbf{5}_j$ states may come from $\mathbf{16}$ and $\overline{\mathbf{16}}$ representations, respectively, in $W_{sym\ breaking}$. We include, however, only those states which mix with $\mathbf{10}_1$.] With this superpotential, we have

$\bar{M}'_d \equiv M[d]$, and the triplet mass matrix, \bar{M}_t , with non-vanishing determinant, includes the terms which mix states in 10_1 with $\bar{5}_i, 5_j$, for $i, j \geq 2$ and the sub-matrix $M[t]$, where the matrix $M[d]$, ($M[t]$) is given by M with Clebschs appropriate for doublets (triplets).

Proton decay in this model is mediated by the Higgses in 10_1 . Hence, the coefficient of the resulting effective dimension 5 baryon violating operators [6,7] is given by the inverse of the triplet mass matrix in the 11 direction, i.e.

$$(M_t^{-1})_{11} \equiv \frac{\det M[t]}{\det \bar{M}_t} = \frac{\det \bar{M}'_d}{\det \bar{M}_t} \times g$$

where $g \equiv \frac{\det M[t]}{\det M[d]}$. We show, in appendix 2, that $g = g(\zeta_1, \dots, \zeta_m)$ is a holomorphic function of the set of U(1) and R invariant products of powers of vevs. If we now define the effective triplet mass by $\tilde{M}_t^{-1} \equiv (M_t^{-1})_{11}$, we obtain the final form for the factor in eqn. (9) – $\frac{\det \bar{M}_t}{\det \bar{M}'_d} = \tilde{M}_t g(\zeta_1, \dots, \zeta_m)$. As a consequence, eqn. (9) becomes

$$\epsilon_3 = F(\zeta_1, \dots, \zeta_m) + \frac{3\tilde{\alpha}_G}{5\pi} \log \left| \frac{\tilde{M}_t}{M_G} \right| + \dots \quad (11)$$

where f and g are absorbed in the function F . We thus find that suppressing the proton decay rate by increasing the ratio $\frac{\tilde{M}_t}{M_G}$ has the effect of increasing the value of ϵ_3 and consequently increasing the value of α_s .³

C. An Example : ϵ_3 in the model of paper II

As an example, for the particular model of paper II, there is only one independent invariant ratio of the GUT scale vevs given by $\zeta = \frac{v\bar{v}}{a_1 a_2}$. In principle, ϵ_3 could depend on an arbitrary function $f(\zeta)$. However, when we put the effective determinants for this model in equation (8) we find

$$\epsilon_3 = \frac{3\tilde{\alpha}_G}{5\pi} \left\{ 21 \log(2) + \log \left| \frac{\tilde{M}_t}{M_G} \right| \right\}. \quad (12)$$

³This observation in the context of minimal SU(5), where in that case \tilde{M}_t is the color triplet Higgs mass, was discussed earlier by Hisano et al. [7].

Remarkably, all dependence of ϵ_3 on the GUT scale vevs has dropped out except for the dependence on \tilde{M}_t . Unfortunately, the large positive constant that appears in the expression means that in order to get ϵ_3 negative, the proton's lifetime would be many orders of magnitude lower than the experimental bound. In addition, changing the Yukawa coupling coefficients of the terms of $W_{sym\ breaking}$ cannot cure this problem. For the most part, changing the Yukawa coupling constants of one of the terms of $W_{sym\ breaking}$ has the result of multiplying the effective determinants of the mass matrices M_γ in eqn. (6), for states γ in a complete SU(5) multiplet, all by the same amount. Thus, this multiplicative factor has no effect on ϵ_3 . Hence, as a consequence of (12), we find that the symmetry breaking sector of the model of paper II is ruled out by the dual constraints coming from the low energy measurement of α_s and the proton lifetime.

Is it possible to find a symmetry breaking sector which has all the U(1) symmetries required for the naturalness of the theory and is consistent with the constraints of α_s and τ_p ? In the next section we describe a new SO(10) SUSY GUT which satisfies all our criteria. In fact it may contain the minimal symmetry breaking sector consistent with the requirements of (1) obtaining the effective fermion mass operators of paper I, (2) “naturalness,” and (3) retaining only those states at energies below M_G , which either have trivial SM charge or are contained in the minimal supersymmetric standard model [MSSM].

III. A COMPLETE SO(10) SUSY GUT

A. The GUT symmetry breaking and Higgs sectors

For this model,

$$\begin{aligned}
 W_{sym\ breaking} = & \frac{1}{M} A'_1 (A_1^3 + \mathcal{S}_3 S A_1 + \mathcal{S}_4 A_1 A_2) \\
 & + A_2 (\psi \bar{\psi} + \mathcal{S}_1 \tilde{A}) + S \tilde{A}^2 \\
 & + S' (S \mathcal{S}_2 + A_1 \tilde{A}) + \mathcal{S}_3 S'^2
 \end{aligned} \tag{13}$$

where the fields $\{A_1, A_2, \tilde{A}, A'_1\}$, $\{S, S'\}$, $\psi, \bar{\psi}$, $\{\mathcal{S}_1, \dots, \mathcal{S}_4\}$ are in the 45, 54, 16, $\bar{16}$ and 1 representations, respectively. [Note, traces and contractions of indices are implicit.] The supersymmetric minimization condition $\frac{\partial W}{\partial A'_1} = 0$ gives four discrete choices for the direction of the vev of A_1 . We will assume that nature chooses the B - L direction. The second term gives \tilde{A} a vev in the X direction, A_2 a vev in the Y direction, and ψ and $\bar{\psi}$ vevs in the right-handed neutrino-like direction (see eqn. (14) below). The third and fourth terms, and the \mathcal{S}_4 subterm of the first term, were added to give mass to all non-MSSM fields which are not in a singlet representation of the standard model gauge group. The last term was added in accordance with our “naturalness” criterion, namely that the theory should be the most general one consistent with the symmetries. The term $\mathcal{S}_3 S'^2$ is consistent with the U(1) and R symmetries of the symmetry breaking sector of the theory that will be discussed below, and we are aware of no other additional symmetry that might exclude this term. Therefore the term must be included.

The above vevs are given by

$$\begin{aligned} \langle A_1 \rangle &= a_1 \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix} \otimes \eta \equiv a_1 \frac{3}{2} (B - L) \\ \langle A_2 \rangle &= a_2 \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & -3/2 & \\ & & & & -3/2 \end{pmatrix} \otimes \eta \equiv a_2 \frac{3}{2} Y \end{aligned} \tag{14}$$

$$\begin{aligned}
\langle \tilde{A} \rangle &= \tilde{a} \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix} \otimes \eta \equiv \tilde{a} \frac{1}{2} X \\
\langle S \rangle &= s \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & -3/2 & \\ & & & & -3/2 \end{pmatrix} \otimes \mathbf{1} \\
\langle \psi \rangle &= v \text{ |SU(5) singlet} \\
\langle \bar{\psi} \rangle &= \bar{v} \text{ |SU(5) singlet}
\end{aligned}$$

where

$$\eta = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \mathbf{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

The vacuum minimization conditions are explicitly

$$\begin{aligned}
a_1^2 &= -s\mathcal{S}_3, & \mathcal{S}_1 \tilde{a} &= \frac{1}{4} v \bar{v} \\
s\mathcal{S}_2 + \frac{2}{5} a_1 \tilde{a} &= 0, & a_2 \mathcal{S}_1 + 2s\tilde{a} &= 0
\end{aligned} \tag{15}$$

Using these equations, the set $\{a_1, a_2, \tilde{a}, v, \bar{v}, \mathcal{S}_4\}$ form a complete set of independent variables.

Two caveats –

- Note at tree level the GUT symmetry breaking vevs are undetermined since the potential in these directions are both supersymmetric and flat. We will not discuss the process

for fixing these vevs in this paper. That analysis must necessarily include supersymmetry breaking effects as well as supergravity and radiative corrections.

- We do not describe the source of supersymmetry breaking in this paper. Soft SUSY breaking operators are included at the GUT scale and renormalized down to the weak scale when making any comparison with the low energy data.

The same doublet-triplet splitting mechanism that was used in paper II can be used in this model. Accordingly, the Higgs sector of the Lagrangian is given by

$$L_{Higgs} = [10_1 A_1 10_2 + \mathcal{S}_5 10_2^2]_F + \frac{1}{M} [z^* 10_1^2]_D \quad (16)$$

where the first term is W_{Higgs} and the second generates a μ term when the F component of the hidden sector field z gets a vev of order $10^{10} GeV$. We then obtain $\mu = \langle F_z \rangle / M$.

1. $U(1)$ and R symmetries of $W_{sym breaking}$ and ϵ_3

$W_{sym breaking}$ has a $[U(1)]^4 \times R$ symmetry as is summarized in table III. Up to arbitrary Yukawa coupling coefficients assumed to be of $O(1)$, $W_{sym breaking}$ is the most general superspace potential consistent with these symmetries. Using $a_1, a_2, \tilde{a}, v, \bar{v}$, and \mathcal{S}_4 as independent variables, the only invariant under a $[U(1)]^4 \times R$ rotation of the vevs is $\zeta = \frac{a_1^4}{a_2^2 \mathcal{S}_4^2}$. After evaluating ϵ_3 explicitly using equation (8) we obtain

$$\epsilon_3 = \frac{3\tilde{\alpha}_G}{10\pi} \left\{ 2 \log \frac{256}{243} - \log \left| \frac{(1 - 25\zeta)^4}{(1 - \zeta)} \right| + 2 \log \left| \frac{\tilde{M}_t}{M_G} \right| \right\}. \quad (17)$$

Before we can check whether the experimental measurement of $\alpha_s(M_Z)$ in this model is consistent with the non-observation of proton decay, we must discuss the fermion mass sector of the theory. The proton lifetime and branching ratios depend crucially on the couplings of the color triplet Higgses to fermions. In a theory of fermion masses, these couplings are related to the Yukawa couplings of fermions to Higgs doublets, *but they are not identical*. For example, the dimension 5 operators responsible for proton decay are given by

$$\begin{aligned} & \frac{1}{M_t} \mathbf{Q} \frac{1}{2} C_{QQ} \mathbf{Q} \mathbf{Q} C_{QL} \mathbf{L} \\ & \frac{1}{M_t} \bar{\mathbf{u}} C_{ue} \bar{\mathbf{e}} \bar{\mathbf{u}} C_{ud} \bar{\mathbf{d}} \end{aligned} \quad (18)$$

where C_{QQ}, C_{QL}, C_{ue} and C_{ud} are 3×3 complex matrices. The matrices C_{QQ} and g_u , the up quark Yukawa matrix, contain the same independent parameters but the $SO(10)$ Clebschs are different. An example is presented in the next section.

B. Fermion mass sector

We take the flavor sector of our model to be that of model 4 in paper I. Preliminary results of Blazek, Carena, Wagner and one of us (S.R.) [12] suggest that this model provides the best fit to the low energy data.

The flavor sector is specified by a particular set of four operators $\{O_{33}, O_{32}, O_{22}, O_{12}\}$. Three of these operators – O_{33}, O_{32} , and O_{12} – are uniquely specified by choosing model 4. On the other hand there are 6 choices for operator O_{22} , labelled (a, \dots, f) , as all give identical entries in the charged fermion Yukawa matrices. As a result we can construct 6 possible models $4(a, \dots, f)$.

The four effective fermion mass operators for model 4c are given by

$$\begin{aligned} O_{33} &= 16_3 10_1 16_3 \\ O_{23} &= 16_2 \frac{A_2}{A} 10_1 \frac{A_1}{A} 16_3 \\ O_{22} &= 16_2 \frac{\tilde{A}}{S_M} 10_1 \frac{A_1}{S_M} 16_2 \\ O_{12} &= 16_1 \left(\frac{\tilde{A}}{S_M}\right)^3 10_1 \left(\frac{\tilde{A}}{S_M}\right)^3 16_2 \end{aligned} \tag{19}$$

The superspace potential for the complete theory above the GUT scale which reproduces model 4c is given by (see fig. 1)

$$\begin{aligned} W_{fermion} = & 16_3 10_1 16_3 + \bar{\Psi}_1 A_1 16_3 + \bar{\Psi}_1 \tilde{A} \Psi_1 + \Psi_1 10_1 \Psi_2 \\ & + \bar{\Psi}_2 \tilde{A} \Psi_2 + \bar{\Psi}_2 A_2 16_2 + \bar{\Psi}_3 A_1 16_2 \\ & + \Psi_3 10_1 \Psi_4 + S_M \sum_{a=3}^9 (\bar{\Psi}_a \Psi_a) \\ & + \bar{\Psi}_4 \tilde{A} 16_2 + \bar{\Psi}_5 \tilde{A} \Psi_4 + \bar{\Psi}_6 \tilde{A} \Psi_5 \end{aligned} \tag{20}$$

$$+ \Psi_6 10_1 \Psi_7 + \bar{\Psi}_7 \tilde{A} \Psi_8 + \bar{\Psi}_8 \tilde{A} \Psi_9 + \bar{\Psi}_9 \tilde{A} 16_1$$

This superpotential is consistent with the symmetries discussed previously with the addition of one new U(1) given in table III. However it is not the most general fermion sector consistent with these symmetries. In fact *one and only one* new operator must be added

$$\bar{\Psi}_6 A_2 16_3. \quad (21)$$

It is easy to see that this operator leads to one new effective operator at the GUT scale when heavy states are integrated out (see fig. 2). The new operator is

$$O_{13} = 16_1 \left(\frac{\tilde{A}}{\tilde{S}_M} \right)^3 10_1 \left(\frac{A_2}{\tilde{S}_M} \right) 16_3. \quad (22)$$

The complete model 4c is thus defined with this new operator and includes the operators in eqn. (19) and (22).

A fit of model 4c to the low energy data [12] for certain ranges of soft SUSY breaking parameters agrees to better than 1σ for all observables. A global χ^2 analysis of models 4, 6 (a - f) (paper I) for all regions of parameter space is now underway [12]. It is already clear, however, that an additional operator such as the 13 operator in eqn. (22) is absolutely necessary to fit the data. Whether this model fits the data better than any other remains to be seen. For example, a different choice of 22 operator results in a different U(1) symmetry and thus by “naturalness,” a distinct theory. In particular, we have checked that for model 4b there are no new effective fermion mass operators generated, while for model 4a the new 13 operator

$$O_{13} = 16_1 \left(\frac{\tilde{A}}{\tilde{S}_M} \right)^3 10_1 \left(\frac{\tilde{A} A_2}{\tilde{S}_M^2} \right) 16_3 \quad (23)$$

is needed.

Finally, consider the matrices relevant for proton decay for the case of model 4c. In particular, the matrix C_{QQ} is given by

$$C_{QQ} = \begin{pmatrix} 0 & C & \frac{1}{3} D e^{i\delta} \\ C & -\frac{1}{2} E e^{i\phi} & \frac{1}{3} B \\ \frac{1}{3} D e^{i\delta} & \frac{1}{3} B & A \end{pmatrix}. \quad (24)$$

This should be compared with the up quark Yukawa matrix in the same model given by

$$g_u = \begin{pmatrix} 0 & C & \frac{1}{3}De^{i\delta} \\ C & 0 & -\frac{4}{3}B \\ -\frac{4}{3}De^{i\delta} & -\frac{1}{3}B & A \end{pmatrix}. \quad (25)$$

where the 7 Yukawa parameters A , B , C , D , E , ϕ and δ are obtained by fitting to quark and lepton masses and the CKM mixing angles [12]. Note that the Clebschs in the two matrices differ by as much as a factor of 4. These Clebschs affect proton decay branching ratios. It is thus important to calculate the branching ratios in models which are consistent with the observed fermion masses and mixing angles.

1. Additional threshold corrections at M_G

The dominant effective operators at the scale M_G are obtained by integrating out states with mass greater than M_G . Higher order corrections to these operators are also obtained. These typically lead to $O(10\%)$ corrections to the leading terms in the Yukawa matrices. Of course the terms in the Yukawa matrices will also receive corrections at one loop. We have neglected these corrections in the following analysis.

2. Neutrino masses

For completeness we include a minimal neutrino mass sector

$$W_{neutrino} = \bar{\psi} \sum_{i=1}^3 16_i N_i \quad (26)$$

where the states N_i , $i = 1, \dots, 3$ are $SO(10)$ singlets. This term has the effect of giving GUT scale Dirac masses to the right-handed neutrinos in the 16's. Thus, the SM left-handed neutrinos are absolutely massless in this theory.

If necessary, left-handed neutrinos can be given masses as described in paper II [10]. However, in order to do so we must either break one linear combination of the $U(1)$ s or

introduce additional SO(10) singlets. In either case, we must then check for “naturalness” and add any operator allowed by the symmetries of the theory.

C. Symmetries

This theory has 5 global U(1) symmetries and a global continuous R symmetry. The charges of most of the states are given in table III. The charges of the other states can easily be derived. We have checked our theory for “naturalness.” We find that only 3 additional operators need to be added to the superpotential –

$$\bar{\psi}_2 A'_1 \psi_1 \mathcal{S}_2, \quad \bar{\psi}_2 A'_1 16_3 \mathcal{S}_3, \quad \bar{\psi}_5 A'_1 \psi_3 \mathcal{S}_3.$$

These three operators have no direct effect on any observable properties since the vev of A'_1 vanishes. With the inclusion of these three operators the total superspace potential for model 4c is “natural” (i.e. no additional operators consistent with these symmetries are allowed) *for all possible powers of the fields*. In addition the theory has a matter reflection symmetry (see Dimopoulos and Georgi, ref. [1]) which forbids dimension 4 baryon or lepton number violating operators.

Some problems, however, remain to be solved in our model. Given the states and symmetries, we find that $f_{\alpha\beta}$, the coefficient of the general gauge kinetic term, is trivial in this model. Thus we have not explicitly included the sector of the theory which generates gaugino masses once SUSY is broken. In addition, the U(1) symmetries of the theory are not sufficient to significantly constrain the Kahler potential. For example, terms such as

$$\frac{1}{M^2} \psi_4^* \mathcal{S}_M^* \tilde{A} 16_2$$

are allowed which mix light generations with heavy states. This term (and others like it) is allowed since it already appears in the structure of the Feynman diagrams of fig. 1. These off diagonal terms in the Kahler potential can affect fermion mass operators as well as introduce flavor changing neutral current processes at low energies [17]. When deriving the effective

theory at M_G we have implicitly assumed that the Kahler potential is universal for all 16s in the theory.

IV. RESULTS FOR $\alpha_S(M_Z)$ VS. PROTON DECAY IN MODEL 4C

We now check whether the new model is consistent with α_s and the proton lifetime. We have calculated the proton decay rate for the three dominant modes — $K^+\bar{\nu}$, $\pi^+\bar{\nu}$, $K^0\mu^+$ where for neutrino modes we sum over the three neutrino species. We have included both gluino and chargino loops, as well as LLLL, LLRR and RRRR operators in our analysis, where L(R) refers to left-handed (right-handed) fermion fields. We used values for the dimensionless Yukawa parameters at M_G which give predictions for fermion masses and mixing angles in excellent agreement with the data [12]. The values for soft SUSY breaking parameters are also consistent with electroweak symmetry breaking and the experimental measurement of $b \rightarrow s + \gamma$. We renormalized the dimensionless (dimensionful) parameters at two (one) loops to low energies in order to make contact with the data. A detailed report on this work is in preparation [13]. For the calculation presented here we take the effective Higgs triplet mass, $\tilde{M}_t = a_1^2/\mathcal{S}_5 = 4 \times 10^{19} GeV$ with $a_1 = M_G \approx 2 \times 10^{16} GeV$ and $\mathcal{S}_5 = 10^{13} GeV$. This corresponds to light Higgs doublets with mass $10^{13} GeV$.

Is it natural to have such light Higgs doublets? Are we populating the GUT desert? To address this question we should compare the Higgs doublet mass with the spectrum of masses for states in the symmetry breaking sector of the theory. These in fact range from $10^{13} - 10^{20} GeV$. Thus we have taken the doublet mass to lie at the lower bound of this GUT scale spectrum. This seems to be the only natural criteria for setting an upper bound on \tilde{M}_t .

The results for proton decay are given in table IV for two different values of soft SUSY breaking parameters and dimensionless couplings [12]. For comparison we also present the branching ratios obtained in a generic minimal SU(5) SUSY GUT from Hisano et al. [7]. The soft SUSY breaking parameters at the GUT scale are for (case A) $M_{1/2} = 250, m_0 = 750, \mu =$

64, $m_{H_u} = 904, m_{H_d} = 1200$ GeV and (case B) $M_{1/2} = 100, m_0 = 3000, \mu = 322, m_{H_u} = 4200, m_{H_d} = 3150$ GeV with for both cases, $A_0 = 0$ and $\tan \beta = 53$. The experimental lower bound on the proton lifetime into the mode $K^+ \bar{\nu}$ is 10^{32} years [18]. Thus these values are consistent with the non-observation of proton decay to date. However, these values are to be considered as *upper limits* on the proton lifetime. In particular, $\tau_p Br^{-1}(p \rightarrow K^+ \bar{\nu})$ scales as $(\frac{\tilde{M}_t}{10^{19} \text{ GeV}} \frac{0.003 \text{ GeV}^3}{\beta})^2$, where β , with values in the accepted range $\beta = (0.003 - 0.03) \text{ GeV}^3$ [19], is a measure of the matrix element of a 3 quark operator between the proton state and the vacuum.

Are these upper bounds on proton decay consistent with the experimental measurement of $\alpha_s(M_Z)$? We have evaluated the expression for ϵ_3 including the additional states with GUT scale masses contained in the fermion mass sector of the theory. These typically shift the value of ϵ_3 by a small amount in the positive direction. We find that for typical values of a_1, a_2 , and \mathcal{S}_4 around the GUT scale, we can obtain ϵ_3 negative for \tilde{M}_t of order 10^{19} GeV. For example, with $a_1 = 2a_2 = 2\mathcal{S}_4 = \frac{\tilde{a}}{3} = M_G$ we have $\zeta = 16$ and $\epsilon_3 \approx -0.030$, which includes a positive contribution of 0.005 from the fermion mass sector.

We have also checked that for these values of the parameters the gauge coupling satisfies one loop perturbativity up to $M = 10^{18}$ GeV with $\alpha_G(M) = 0.39$. Note that above the GUT scale we use the threshold boundary condition

$$\alpha_G^{-1}(M_G) = \tilde{\alpha}_G^{-1} + \Delta_2(M_G)$$

.

V. CONCLUSION

We have presented a new complete SO(10) SUSY GUT. This theory has several interesting features. The superspace potential is “natural” to all orders in the fields. It contains what may possibly be the minimal GUT symmetry breaking sector necessary to obtain the desired adjoint vevs consistent with (1) “naturalness” and (2) fermion masses.

We have also shown that one loop GUT scale threshold corrections are a significant constraint on the GUT symmetry breaking sector of the theory. This constraint, for example, is sufficient to rule out the model of paper II [10]. The one loop threshold corrections relate the prediction for $\alpha_s(M_Z)$ to the proton lifetime.

Finally we have calculated one loop GUT scale threshold corrections to gauge couplings and proton decay rates to different final states in the new model. The results are consistent with the low energy measurement of $\alpha_s(M_Z)$. The proton decay branching ratios provide a powerful test for theories of fermion masses. There is reasonable hope that new results from SuperKamiokande or Icarus could confirm or rule out these theories.

It would be presumptuous of us to conclude without discussing some of the open questions. We have not discussed the origin of SUSY breaking nor how it feeds into the visible sector with the one exception of the μ term which we have nominally considered. We also do not discuss what determines the GUT vevs. At tree level, in the globally supersymmetric theory considered here and neglecting SUSY breaking, these are flat directions of the potential. Finally, and perhaps most seriously, the symmetries discussed in this paper do not significantly constrain the Kahler potential. Flavor mixing in the Kahler potential could lead to dangerous flavor changing neutral current processes. We have *assumed* the trivial universal Kahler potential in our analysis.

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APPENDIX 1

Proof of equation (6)

We define the GUT scale M_G as the point where $\tilde{\alpha}_G \equiv \alpha_1(M_G) = \alpha_2(M_G)$. This means that $\Delta_1(M_G) = \Delta_2(M_G)$. We then define the relative shift in $\alpha_3(M_G)$ by

$$\begin{aligned}\epsilon_3 &\equiv (\alpha_3(M_G) - \tilde{\alpha}_G)/\tilde{\alpha}_G \\ &= \alpha_3(M_G) (\Delta_3 - \Delta_1|_{M_G})\end{aligned}\tag{27}$$

We then have

$$\Delta_3 - \Delta_1|_{M_G} = \frac{1}{2\pi} \left(\sum_{\gamma} (b_3^{\gamma} - b_1^{\gamma}) \log |\det' M_{\gamma}| - \sum_{\gamma} (b_3^{\gamma} - b_1^{\gamma}) n_{\gamma} \log M_G \right)\tag{28}$$

where n_{γ} = the mass dimension of $\det' M_{\gamma}$ and $\det' M_{\gamma}$ is defined in eqn. (7), except for $\det' M_d$ where it will be convenient to define \tilde{M}_d by $\det \tilde{M}_d = M_G \det M'_d$, and $\det' M_d = \det \tilde{M}_d$. This redefinition does not affect eqn. (6). Note, the matrix \tilde{M}_d is defined such that $n_t = n_d$.

In addition,

$$\Delta_1|_{M_G} = \Delta_2|_{M_G}$$

which implies

$$\sum_{\gamma} (b_1^{\gamma} - b_2^{\gamma}) \log |\det' M_{\gamma}| = \left(\sum_{\gamma} (b_1^{\gamma} - b_2^{\gamma}) n_{\gamma} \right) \log M_G\tag{29}$$

Substituting for $\log M_G$ we obtain

$$\begin{aligned}\epsilon_3 &\approx \frac{\tilde{\alpha}_G}{2\pi} \left\{ \sum_{\gamma} (b_3^{\gamma} - b_1^{\gamma}) \log |\det' M_{\gamma}| - \frac{\sum_{\gamma} (b_3^{\gamma} - b_1^{\gamma}) n_{\gamma}}{\sum_{\gamma} (b_1^{\gamma} - b_2^{\gamma}) n_{\gamma}} \sum_{\gamma} (b_1^{\gamma} - b_2^{\gamma}) \log |\det' M_{\gamma}| \right\} \\ &= \frac{1}{2\pi} \frac{1}{c_{12}} \sum_{\gamma} (b_1^{\gamma} c_{23} + b_2^{\gamma} c_{31} + b_3^{\gamma} c_{12}) \log |\det' M_{\gamma}|\end{aligned}\tag{30}$$

where $c_{ij} = \sum_{\gamma} (b_i^{\gamma} - b_j^{\gamma}) n_{\gamma}$. To evaluate the c_{ij} s, define n_{54} , n_{45} and n_{10} to be the number of 54, 45, and 10 representations in the theory, respectively, and n_{16} and $n_{\overline{16}}$ to be the number of supermassive 16 and $\overline{16}$ representations, respectively. For any SO(10) model built with

only 1, 10, 16, $\overline{16}$, 45, and 54 representations and one pair of light Higgs doublets, we have

$$\begin{aligned}
n_{16} &= n_{\overline{16}} \\
n_g = n_w &= n_{45} + n_{54} \\
n_x &= n_{45} + n_{54} - 3 \\
n_u = n_e &= n_{45} + n_{16} - 3 \\
n_s = n_\sigma &= n_{54} \\
n_q &= n_{45} + n_{54} + n_{16} - 3 \\
n_d = n_t &= n_{10} + n_{16}
\end{aligned}$$

Evaluating c_{23} explicitly

$$\begin{aligned}
c_{23} &= (4n_s + 3n_q + 2n_w + 3n_x + n_d) - (5n_s + 2n_q + n_u + 3n_g + 2n_x + n_t) \\
&= \{4(n_{54}) + 3(n_{54} + n_{45} + n_{16} - 3) + 2(n_{54} + n_{45}) + \\
&\quad 3(n_{54} + n_{45} - 3) + (n_{10} + n_{16}) \\
&\quad - [5(n_{54}) + 2(n_{54} + n_{45} + n_{16} - 3) + (n_{45} + n_{16} - 3) + 3(n_{54} + n_{45}) + \\
&\quad 2(n_{54} + n_{45} - 3) + (n_{10} + n_{16})]\} \\
&= -3
\end{aligned} \tag{31}$$

Similarly

$$\begin{aligned}
c_{31} &= 9 \\
c_{12} &= -6
\end{aligned} \tag{32}$$

Thus, the c_{ij} s are completely independent of the number of fields in any theory built with 1s, 10s, 16s, $\overline{16}$ s, 45s, and 54s. Plugging eqns. (31) and (32) into eqn. (30), eqn. (6) readily follows.

APPENDIX 2

U(1) symmetries and the dependence of ϵ_3 on GUT scale vevs

In this appendix we prove that the contribution to ϵ_3 from the GUT symmetry breaking sector is only a function of U(1) and R invariant products of powers of vevs. The proof relies on two facts:

1. the effective determinants of mass matrices are holomorphic functions of the symmetry breaking vevs, and
2. the effective determinants have simple phase rotations under U(1) and R symmetry transformations.

Note, since the effective determinants are independent of the conjugates of vevs, the U(1) and R invariance of ϵ_3 is very restrictive.

We first discuss the case for mass matrices which do *not* include vector multiplets, followed immediately by the case for mass matrices including vector multiplets. Note, in the first case the mass matrices themselves are, by construction, holomorphic functions of vevs. This is however not true for the latter case which is why it requires a separate discussion.

Consider a general superspace potential $W(\Phi_1, \Phi_2, \dots, \Phi_N)$ whose superfields rotate under a U(1) symmetry as

$$\Phi_j \xrightarrow{\theta} e^{iQ_j\theta} \Phi_j$$

By defining the shifted fields $\hat{\Phi}_i \equiv \Phi_i - \langle \Phi_i \rangle$ and expanding the superspace potential about $\langle \Phi \rangle$ we can find the fermion mass matrices.

$$W(\hat{\Phi}_1 + \langle \Phi_1 \rangle, \dots) = \dots + \sum_{\gamma} \psi_{\gamma} m_{\gamma}(\langle \Phi_1 \rangle, \langle \Phi_2 \rangle, \dots) \psi_{\gamma} + \dots \quad (33)$$

Now consider what would happen if the superfields received a different vacuum expectation value,

$$\langle \Phi_j \rangle^{\text{new}} = e^{iQ_j\theta} \langle \Phi_j \rangle^{\text{old}}$$

Under this change, the mass matrices would change.

$$W(\hat{\Phi}_1 + e^{iQ_1\theta} \langle \Phi_1 \rangle, \dots) = \dots + \sum_{\gamma} \psi_{\gamma} m_{\gamma}^{\theta} \psi_{\gamma} + \dots \quad (34)$$

where

$$m_{\gamma}^{\theta} \equiv m_{\gamma}(e^{iQ_1\theta} \langle \Phi_1 \rangle, e^{iQ_2\theta} \langle \Phi_2 \rangle, \dots)$$

However, if we rotate the shifted fields $\hat{\Phi}_j$ by $e^{iQ_j\theta}$, the superspace potential will be invariant under the combined rotation of $\hat{\Phi}$ and $\langle \Phi \rangle$.

$$\begin{aligned}
& W(e^{iQ_1\theta}\hat{\Phi}_1 + e^{iQ_1\theta}\langle\Phi_1\rangle, \dots, e^{iQ_N\theta}\hat{\Phi}_N + e^{iQ_N\theta}\langle\Phi_N\rangle) \\
&= \dots + \sum_{\gamma} \psi_{\gamma} \begin{pmatrix} e^{iQ_1\theta} & & \\ & e^{iQ_2\theta} & \\ & & \ddots \end{pmatrix} m_{\gamma}^{\theta} \begin{pmatrix} e^{iQ_1\theta} & & \\ & e^{iQ_2\theta} & \\ & & \ddots \end{pmatrix} \psi_{\gamma} + \dots \\
&= W(\hat{\Phi}_1 + \langle\Phi_1\rangle, \dots, \hat{\Phi}_N + \langle\Phi_N\rangle) \\
&= \dots + \sum_{\gamma} \psi_{\gamma} m_{\gamma} \psi_{\gamma} + \dots
\end{aligned} \tag{35}$$

Therefore, m_{γ}^{θ} rotates in a very simple way.

$$m_{\gamma}^{\theta} = \begin{pmatrix} e^{-iQ_1\theta} & & \\ & e^{-iQ_2\theta} & \\ & & \ddots \end{pmatrix} m_{\gamma} \begin{pmatrix} e^{-iQ_1\theta} & & \\ & e^{-iQ_2\theta} & \\ & & \ddots \end{pmatrix} \tag{36}$$

Thus,

$$\det m_{\gamma}^{\theta} = e^{-2i\theta \sum Q} \det m_{\gamma} \tag{37}$$

where the sum is over all fields that have columns in the mass matrix.

Similar arguments can show that under an R symmetry rotation, $\langle\Phi\rangle \rightarrow e^{iQ\theta}\langle\Phi\rangle$, m_{γ}^{θ} is equal to

$$e^{iQ_W\theta} \begin{pmatrix} e^{-iQ_1\theta} & & \\ & e^{-iQ_2\theta} & \\ & & \ddots \end{pmatrix} m_{\gamma} \begin{pmatrix} e^{-iQ_1\theta} & & \\ & e^{-iQ_2\theta} & \\ & & \ddots \end{pmatrix} \tag{38}$$

where Q_W is the charge of the superspace potential under the R symmetry. Therefore,

$$\det m_{\gamma}^{\theta} = e^{iQ_W N\theta - 2i\theta \sum Q} \det m_{\gamma} \tag{39}$$

where $N = \dim m_{\gamma}$.

The situation is a bit more complicated for mass matrices which receive contributions from vector multiplets. The proof that the determinants and hence the effective determinants have simple phase rotations under the U(1) and R symmetry transformations can readily be

extended to these mass matrices. However, since the entries in the gaugino-chiral fermion mixing rows are actually the complex conjugates of vevs, the determinants of these matrices are not holomorphic. However, in the following we prove that the *effective determinants* of these mass matrices, which include vector multiplets, *are* in fact holomorphic functions of the vevs.

Since the would-be Goldstone fermion states are perpendicular to the massive chiral fermion states, any mass matrix containing gaugino-chiral fermion mixing can be written in the form

$$\begin{pmatrix} 0 & x_1^* & x_2^* & x_3^* & \cdots & x_N^* \\ \bar{x}_1^* & c_{11} & c_{12} & c_{13} & \cdots & c_{1N} \\ \bar{x}_2^* & c_{21} & c_{12} & c_{23} & \cdots & c_{2N} \\ \bar{x}_3^* & c_{31} & c_{12} & c_{33} & \cdots & c_{3N} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar{x}_N^* & c_{N1} & c_{N2} & c_{N3} & \cdots & c_{NN} \end{pmatrix} \quad (40)$$

with $\sum_j c_{ij}x_j = 0$ for all i , $\sum_i c_{ij}\bar{x}_i = 0$ for all j , and the c_{ij} s, x s, and \bar{x} s are functions of the vevs but not of their conjugates. The rows and columns of the mass matrix can be rearranged so that x_N and \bar{x}_N are not zero. However, the determinant of this matrix can be reduced by elementary row and column operations. Namely, by adding to the last column the second column multiplied by $\frac{x_1}{x_N}$ plus the third column multiplied by $\frac{x_2}{x_N}$, and so forth, the determinant of the mass matrix becomes

$$\begin{vmatrix} 0 & x_1^* & x_2^* & \cdots & x_{N-1}^* & x_N^* + \frac{x_1^*x_1}{x_N} + \cdots + \frac{x_{N-1}^*x_{N-1}}{x_N} \\ \bar{x}_1^* & c_{11} & c_{12} & \cdots & c_{1,N-1} & 0 \\ \bar{x}_2^* & c_{21} & c_{12} & \cdots & c_{2,N-1} & 0 \\ \bar{x}_3^* & c_{31} & c_{12} & \cdots & c_{3,N-1} & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \bar{x}_N^* & c_{N1} & c_{N2} & \cdots & c_{N,N-1} & 0 \end{vmatrix}$$

Doing the analogous operation on the rows, the determinant becomes

$$\begin{vmatrix} 0 & x_1^* & x_2^* & \cdots & x_{N-1}^* & \frac{1}{x_N} \sum_k^N x_k^* x_k \\ \bar{x}_1^* & c_{11} & c_{12} & \cdots & c_{1,N-1} & 0 \\ \bar{x}_2^* & c_{21} & c_{12} & \cdots & c_{2,N-1} & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \bar{x}_{N-1}^* & c_{N-1,1} & c_{N-1,2} & \cdots & c_{N-1,N-1} & 0 \\ \frac{1}{\bar{x}_N} \sum_k^N \bar{x}_k^* \bar{x}_k & 0 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

Thus, the determinant is equal to

$$\begin{aligned} & \frac{\sum_k^N x_k^* x_k}{x_N} \frac{\sum_k^N \bar{x}_k^* \bar{x}_k}{\bar{x}_N} \begin{vmatrix} c_{11} & \cdots & c_{1,N-1} \\ \vdots & \ddots & \vdots \\ c_{N-1,1} & \cdots & c_{N-1,N-1} \end{vmatrix} \\ &= \left(\sum_k^N x_k^* x_k \right) \left(\sum_k^N \bar{x}_k^* \bar{x}_k \right) f(\text{vevs}) \end{aligned} \quad (41)$$

where f is a holomorphic function of the vevs. By setting $\bar{v} = v$, \bar{x}_i will equal x_i for all i and the mass of the vector multiplet is $\sqrt{\sum_k^N x_k^* x_k}$. Therefore, the determinant is equal to $M_{\text{vector multiplet}}^4$ times f and the effective determinant is just the function f . Note, $M_{\text{vector multiplet}}$ is thus always canceled from the denominator of the effective determinant (eqn. 6) and no conjugates of vevs can appear in the effective determinant. Thus, the effective determinants for mass matrices containing gauginos are holomorphic functions of the vevs; just as those discussed earlier for mass matrices which do not have gaugino-chiral fermion mixing entries.

These simple transformation properties of the mass matrices, under $U(1)$ rotations of the vevs, have significant consequences for the form of ϵ_3 . Consider the following expression entering ϵ_3 (see eqn. 8) —

$$\begin{aligned} & \frac{3}{2} \log \det' \frac{M_g}{M_G} - \frac{3}{2} \log \det' \frac{M_w}{M_G} + \frac{33}{10} \log \det' \frac{M_s}{M_G} - \frac{21}{10} \log \det' \frac{M_\sigma}{M_G} \\ & + \frac{9}{10} \log \det' \frac{M_u}{M_G} + \frac{3}{10} \log \det' \frac{M_e}{M_G} - \frac{6}{5} \log \det' \frac{M_q}{M_G} \end{aligned} \quad (42)$$

It is now easy to show that it is invariant under the U(1) and R symmetries. Namely, the U(1) charge of the determinant of M_w (eqn. 37) is equal to the charge of the determinant of M_g which is equal to -2 times the sum of U(1) charges of all fields in the 24 representation of SU(5). Therefore, the U(1) rotation of $\det M_g$ will cancel the rotation of $\det M_w$ in expression, eqn. (42). In addition, we note that the U(1) charges of the effective determinants of $\{M_s, M_\sigma\}$, $\{M_u, M_e\}$, and M_q equal -1 times the sum of U(1) charges of all fields in the $\{15 \text{ and } \overline{15}\}$; $\{10 \text{ and } \overline{10}\}$, and $\{10, 15, \overline{10} \text{ and } \overline{15}\}$ representations of SU(5), respectively. Therefore, the U(1) rotation of $\det' M_q$ is canceled by the rotations of the effective determinants of M_u, M_e, M_s , and M_σ in expression, eqn. (42). Similar arguments show that the expression in eqn. (42) is invariant under an R symmetry rotation. Thus finally we arrive at the conclusion that the expression in eqn. (42) is invariant under the U(1) and R symmetries of $W_{sym. breaking}$ for any superspace potential built with 1, 10, 16, $\overline{16}$, 45, and 54 representations of SO(10). Moreover, since expression, eqn. (42), is holomorphic, the contribution of the GUT symmetry breaking sector to ϵ_3 is only a function of U(1) and R invariant products of powers of vevs.

The same is not true for the contributions from either the Higgs or fermion mass sectors. This is because both the Higgs and fermion mass sectors contain massless states that must be projected out of the mass matrices before the effective determinants are taken. After this projection, the determinants of the resulting mass matrices are no longer holomorphic functions of the vevs. Nevertheless for the Higgs sector we can prove a similar but limited result, namely, that $g \equiv \frac{\det M[t]}{\det M[d]} = g(\zeta_1, \dots, \zeta_m)$; i.e., g is a function of U(1) invariants only. By eqns. (37) and (39), we see that $\det M[t]$ and $\det M[d]$ transform in the same way under the U(1) and R symmetries; therefore the ratio is invariant.

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TABLES

TABLE I. SU(5) decomposition of SO(10) fields in our theory

$$\begin{aligned}
54 &\rightarrow 24 + 15 + \overline{15} \\
45 &\rightarrow 24 + 10 + \overline{10} + 1 \\
16 &\rightarrow 10 + \overline{5} + 1 \\
10 &\rightarrow 5 + \overline{5}
\end{aligned}$$

TABLE II. Notation used for states in different charge sectors

SU(3) \times SU(2) \times U(1) representation	name	appears in SU(5) representation
(8, 1, 0)	g	24
(1, 3, 0)	w	24
(3, 2, $-\frac{5}{3}$); ($\overline{3}$, 2, $\frac{5}{3}$)	$x; \overline{x}$	24
(3, 1, $\frac{4}{3}$); ($\overline{3}$, 1, $-\frac{4}{3}$)	$u; \overline{u}$	$\overline{10}; 10$
(1, 1, -2); (1, 1, 2)	$e; \overline{e}$	$\overline{10}; 10$
(3, 2, $\frac{1}{3}$); ($\overline{3}$, 2, $-\frac{1}{3}$)	$q; \overline{q}$	10, 15; $\overline{10}, \overline{15}$
(6, 1, $-\frac{4}{3}$); ($\overline{6}$, 1, $\frac{4}{3}$)	$s; \overline{s}$	15; $\overline{15}$
(1, 3, 2); (1, 3, -2)	$\sigma; \overline{\sigma}$	15; $\overline{15}$
(3, 1, $-\frac{2}{3}$); ($\overline{3}$, 1, $\frac{2}{3}$)	$t; \overline{t}$	5; $\overline{5}$
(1, 2, 1); (1, 2, -1)	$d; \overline{d}$	5; $\overline{5}$

TABLE III. U(1) and R charges of the new model

field	U(1) charge	field	U(1) charge
A_1	(1, 0, 0, 0, 0)	A'_1	(-3, 0, 0, 0, 0)
A_2	(0, 1, 0, 0, 0)	\tilde{A}	(0, 0, 1, 0, 0)
S	(0, 0, -2, 0, 0)	S'	(-1, 0, -1, 0, 0)
ψ	(0, -1, 0, 1, 0)	$\bar{\psi}$	(0, 0, 0, -1, 0)
\mathcal{S}_1	(0, -1, -1, 0, 0)	\mathcal{S}_2	(1, 0, 3, 0, 0)
\mathcal{S}_3	(2, 0, 2, 0, 0)	\mathcal{S}_4	(2, -1, 0, 0, 0)
\mathcal{S}_5	(2, 0, 0, 0, -4)	16_3	(0, 0, 0, 0, 1)
16_2	(-1, -1, 2, 0, 1)	16_1	(-2, -5, 7, 0, 1)
10_1	(0, 0, 0, 0, -2)	10_2	(-1, 0, 0, 0, 2)
z	(0, 0, 0, 0, -4)	N_3	(0, 0, 0, 1, -1)
N_2	(1, 1, -2, 1, -1)	N_1	(2, 5, -7, 1, -1)

All fields, except A'_1 and z , have R charge 1. A'_1 (z) has R charge 0 (2). The superpotential has R charge 3. The first 4 charges listed above contribute to states in the GUT symmetry breaking sector.

TABLE IV. Proton decay results for model 4c

	A	B	C
$\tau_p Br^{-1}(p \rightarrow K^+ \bar{\nu})/(10^{32} yrs.)$	66	3000	
$\frac{\Gamma(p \rightarrow \pi^+ \bar{\nu})}{\Gamma(p \rightarrow K^+ \bar{\nu})}$	0.28	1.18	0.5
$\frac{\Gamma(p \rightarrow K^0 \mu^+)}{\Gamma(p \rightarrow K^+ \bar{\nu})}$	0.0040	0.0045	0.0007

For $\tilde{M}_t = 10^{19} GeV, \beta = 0.003 GeV^3$. Note, $\tau_p Br^{-1}(p \rightarrow K^+ \bar{\nu})$ scales as $(\frac{\tilde{M}_t}{10^{19} GeV} \frac{0.003 GeV^3}{\beta})^2$. The soft SUSY breaking parameters for case A and B are described in the text. In C, we show the results from Hisano et al. [7] for comparison. Those results are from a generic minimal SU(5) SUSY GUT.

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